Resummed pole mass, static potential, and $\Upsilon(1S)$ spectrum

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Abstract

Employing the heavy quark pole mass and static potential that were obtained from resummation of the renormalon-caused large order behavior to all orders, and matching the resummed static potential to the long distance Cornell potential we obtain a new estimate of the bottom quark mass from $\Upsilon(1S)$ system. We point out that the pure perturbative QCD approach to $\Upsilon(1S)$ spectrum should have a sizable systematic error of about 100 MeV, due to the absence of the long distance confining potential, and that the incorporation of the latter is essential for accurate determination of the ground state energy. We also recalculate the nonperturbative Leutwyler-Voloshin effect using a resummed octet potential matched to a confining linear potential, and obtain a result that is substantially smaller than the estimates based on Coulombic potentials. The hyperfine splitting is estimated to be 50 ± 8 MeV at $\alpha_s(M_z) = 0.118$.

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I.   INTRODUCTION

The heavy quarkonium is a quantum chromodynamic (QCD) system that essentially depends on the two fundamental parameters, the heavy quark mass and strong coupling constant. The precise experimental determination of the spectra of heavy quarkonium such as bottomonium provides a good opportunity to test QCD as well as determine the fundamental parameters.

Traditionally, the heavy quarkonium was investigated primarily within the phenomenological potential models that proved very successful. Nevertheless, the ambiguities in the potential models forbade a precise connection to QCD, and thus an accurate determination of the QCD parameters in this approach was not feasible. The recent approaches to quarkonium system is based on effective theories such as the potential NRQCD (pNRQCD) [1]. In pNRQCD Lagrangian the heavy quark mass, which appears in its most natural form as a pole mass, and the interquark potentials are matching coefficients, which can be calculated, formally, in perturbative QCD (pQCD).

For the bottomonium the first energy eigenstates are expected to be largely determined by the interquark potentials at short distances which can be calculated in pQCD. If the nonperturbative effects are indeed small then these first states are essentially perturbative, and can open a window for testing pQCD as well as determining the QCD parameters. Crucial for the success of this approach is that the nonperturbative effects be small and the potentials at short distances as well as the pole mass be calculable reliably within pQCD. It is, however, well known that because of the infrared (IR) renormalons the perturbative expansions for the pole mass as well as the static potential, which are known to next-next leading order (NNLO), show bad convergence even at scales where pQCD should be applicable.

Besides the apparent convergence problem the IR renormalons pose a more fundamental problem of defining the infrared-sensitive quantities. Let us focus first on the pole mass for the moment, but the discussion in the following applies equally well to the static potential. The renormalon divergence in the perturbative expansion of the pole mass causes an intrinsic uncertainty of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$, and this results in unavoidable ambiguity in defining the pole mass. Nevertheless, a pole mass can be well defined by Borel resummation. In Borel resummation the renormalon ambiguity appears as an ambiguity in taking the integration contour over the renormalon singularity on the Borel plane. Despite this ambiguity one can define a pole mass simply by taking the principal value prescription for the Borel integral. We call the pole mass defined this way the BR mass [2, 3]. The BR mass, by definition, has exactly the same perturbative expansion as the pole mass in pQCD, and thus the coefficients suffer from the same renormalon divergence, but nonetheless it is well-defined. Obviously, the BR mass is not a short-distance mass, and in this respect it is unique among the various heavy quark mass definitions. The ambiguity in defining the pole mass cancel out in physical obeservables, for instance in heavy quarkonium Hamiltonian the ambiguities in the pole mass and the static potential cancel each other, leaving the Hamiltonian free from renormalon divergence [4, 5, 6]. It is noted that we focus, throughout the paper, exclusively on the first IR renormalon since it is overwhelmly dominant in the divergence of the perturbative coefficients of the pole mass as well as the static potential.

Although the Borel resummation formally resolves the renormalon problem, practicing it is another matter since the calculation of the Borel integral to an accuracy better than the renormalon ambiguity demands the Borel transform be known precisely in the domain

containing the origin as well as the first IR renormalon singularity. Although the behavior of the Borel transform about the origin can be obtained by the ordinary perturbative expansions, obtaining its behavior from the origin to the renormalon singularity with the known first terms of perturbation is nontrivial, since it requires accurate knowledge on the renormalon-caused large order behavior. This difficulty of Borel resummation led in the past to the abandonment of an independent notion of infrared sensitive quantities such as the pole mass, and to the adoption of the short-distance quantities instead [7].

Since the difficulty of achieving a manageable definition of an infrared sensitive quantity arises from the difficulty of describing the Borel transform about the renormalon singularity, it is worthwhile to look at this problem more carefully. As is well known, the exact nature of the renormalon singularity can be determined by renormalization group (RG) equation, and the only missing information about the renormalon singularity is the residue that determines the overall normalization constant of the large order behavior. Although the residue cannot be calculated exactly, there is a perturbation scheme in which the residue can be expressed as a convergent series that can be calculated using the ordinary perturbative expansions [8, 9]. The scheme, when applied to QCD observables, yields residues that show surprisingly good convergence even with the first terms of the perturbative expansions, which indicates the residues can be calculated fairly well from the low order perturbative calculations. It was observed recently that this method works particularly well for the pole mass and the static potential because of the overwhelming dominance of the first IR renormalon in these quantities [10, 11, 12].

With the residue known approximately, one can obtain a good description of the Borel transform in the region that contains the origin and the renormalon singularity through an interpolation of the perturbative expansions about the origin and the functional behavior about the renornalom singularity. A particular prescription of the interpolation called *bilocal expansion* was proposed in [13], which incorporates the correct nature of the renormalon singularity and gives, by definition, exactly the same perturbative expansion about the origin as the true Borel transform. The Borel resummation employing this interpolating Borel transform then sums the renormalon-caused large order behavior to all orders, giving an all-order definition for the infrared sensitive quantity. The pole mass and static potential computed in this scheme show remarkable convergence under the bilocal expansion, and in the case of the latter an excellent agreement was obtained with the lattice potential [13]. With the Borel summation technique we thus have a well-defined pole mass that has exactly the same perturbative expansion as the pole mass in pQCD, but nonetheless converges rapidly under the bilocal expansion.

In this paper we apply the Borel resummed pole mass and the static potential to $\Upsilon(1S)$ system to determine the bottom quark mass and the hyperfine splitting. Our approach is unique in that it is not based on short-distance quantities. An useful feature of employing the all-order quantities is that a natural scale separation occurs in a multi-scale system such as the quarkonium. In the schemes that employ short-distance quantities the large and small scales mix invariably in the perturbative calculations, causing large perturbative coefficients that render the calculations less reliable. Since in our scheme the pole mass and the potential can be resummed at scales that are optimal for each quantities, for example, the heavy quark mass and the inverse of the interquark distance, respectively, the Hamiltonian in our approach does not suffer from the scale mixing problem. This is a feature that differentiates our scheme from those based on short-distance quantities. There are other advantages, for instance, having a well-defined, stand-alone potential makes it easy to merge the resummed

potential at short distance to the phenomenological long distance confining potential.

The paper is organized as follows. In Sec. II, we define the Hamiltonian in BR scheme that is obtained, at short and intermediate distances, rigorously from pQCD plus resummation and at intermediate and long distance by matching the long distance Cornell potential to the BR potential. In Sec. III, we show that the incorporation of the long distance confining potential is important for $\Upsilon(1S)$ and the computations based on pure pQCD should contain sizable systematic error. In Sec. IV, we recalculate the Leutwyler-Voloshin type nonperturbative effects employing the resummed singlet and octet potentials and show that they are sufficiently small enough to be negligible for the 1S state. In Sec. V we give our estimate of the $\overline{\text{MS}}$ bottom quark mass and hyperfine splitting, and in Sec. VI give a summary.

II. HAMILTONIAN

The pNRQCD Lagrangian for heavy quarkonium contains singlet as well as octet fields and ultrasoft gluons which represent the dynamical degrees of freedom at scales below the typical heavy-quark momentum. Ignoring for the moment the interactions between the singlet and octet fields, the pNRQCD Hamiltonian for singlet quarkonium in on-shell scheme is given by [14]

$$H = H_0 + H',\tag{1}$$

where

$$H_0 = 2\,m_{pole} - \frac{\nabla^2}{m_{pole}} + V^{(0)}(r)\,, \qquad H' \equiv \sum_{n=1} \frac{V^{(n)}}{m_{pole}^n}\,.\tag{2}$$

For the 1S state H', in the leading order, reads [15, 16]

$$H' = V_{NA} + V_{hf} + V_{SI}\,,\tag{3}$$

with the nonabelian and hyperfine terms given by

$$V_{NA} = -\frac{C_A C_F \alpha_s(\mu)^2}{2m_b r^2}\,, \qquad V_{hf} = \frac{8\pi C_F \alpha_s(\mu)}{3m_b^2}\delta^3(\vec{r})\,,\tag{4}$$

and the spin–independent operator by

$$V_{SI} = -\frac{\vec{p}^4}{4m_b^3} + \frac{\pi C_F \alpha_s(\mu)}{m_b^2}\,\delta^3(\vec{r}) - \frac{C_F \alpha_s(\mu)}{m_b^2 r}\vec{p}^2\,.\tag{5}$$

We shall now focus on the zeroth order Hamiltonian H_0. As mentioned the pole mass m_{pole} and singlet potential $V^{(0)}$ in H_0 are ill-defined in pQCD because of the renormalon divergences in the perturbative matching relations [17, 18]

$$m_{pole} = m_{\overline{\text{MS}}}(1 + \sum_{n=0}^{\infty} p_n(m_{\overline{\text{MS}}}, \mu)\alpha_s(\mu)^{n+1})\,,$$

$$V^{(0)}(r) = -\frac{1}{r}\sum_{n=0}^{\infty} V_n^{(0)}(r, \mu)\alpha_s(\mu)^{n+1}\,,\tag{6}$$
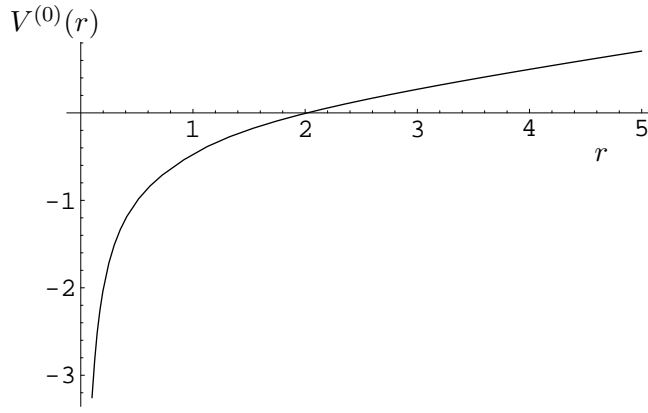
FIG. 1: The improved potential $V^{(0)}(r)$ obtained by matching the BR potential and Cornell potential.

where $\alpha_s(\mu)$ denotes the strong coupling constant.

The Hamiltonian in BR scheme can be obtained by substituting $m_{pole}, V^{(0)}(r)$ with the Borel resummed quantities $m_{\rm BR}, V_{\rm BR}^{(0)}(r)$ which are defined by [2, 3, 12]

$$
\begin{aligned}
m_{\rm BR}(r) &= m_{\overline{\rm MS}}\left(1 + Re\left[\frac{1}{\beta_0}\int_{0+i\epsilon}^{\infty+\epsilon} e^{-b/\beta_0\alpha_s(m_{\overline{\rm MS}})}\tilde{m}(b, m_{\overline{\rm MS}})d\,b\right]\right), \\
V_{\rm BR}^{(0)}(r) &= Re\left[\frac{1}{\beta_0}\int_{0+i\epsilon}^{\infty+\epsilon} e^{-b/\beta_0\alpha_s(1/r)}\tilde{V}^{(0)}(b, 1/r)d\,b\right].
\end{aligned}
\tag{7}
$$

The Borel transforms in these equations have perturbative expansions about the origin,

$$
\begin{aligned}
\tilde{m}(b, m_{\overline{\rm MS}}) &= \sum_{n=0}\frac{p_n(m_{\overline{\rm MS}}, m_{\overline{\rm MS}})}{n!}\left(\frac{b}{\beta_0}\right)^n, \\
\tilde{V}^{(0)}(b, 1/r) &= \sum_{n=0}\frac{V_n^{(0)}(r, 1/r)}{n!}\left(\frac{b}{\beta_0}\right)^n,
\end{aligned}
\tag{8}
$$

where β_0 denotes the one loop coefficient of the QCD β function, and about the renormalon singularity:

$$
\begin{aligned}
\tilde{m}(b, m_{\overline{\rm MS}}) &= \frac{C_m}{(1-2b)^{1+\nu}}\left[1 + c_1(1-2b) + c_2(1-2b)^2 + \ldots\right] + \text{(analytic part)}, \\
\tilde{V}^{(0)}(b, 1/r) &= \frac{C_v}{(1-2b)^{1+\nu}}\left[1 + c_1(1-2b) + c_2(1-2b)^2 + \ldots\right] + \text{(analytic part)},
\end{aligned}
\tag{9}
$$

where ν and c_1, c_2 are exactly known constants. The renormalon residues C_m, C_v can be computed perturbatively following Refs. [8, 9]. Combining the expansions Eq. (8) and Eq. (9) in bilocal expansions, a good description of the Borel transforms can be obtained from the known NNLO perturbations in Eq. (6). For details we refer the readers to [2, 3, 12].

Note that in the Borel summations (7) we have chosen separate scales, $\mu = m_{\overline{\rm MS}}$ for the pole mass and $\mu = 1/r$ for the potential, which are, respectively, the optimal scales for the perturbative expansions (6). Since the renormalon ambiguities must be RG invariant we can choose the renormalization scales independently, without spoiling the cancellation of the renormalon ambiguities between the pole mass and the static potential. This is the crucial feature of our Borel resummation approach.

Although the Borel resummed potential is expected to give a good description at short distance it will fail at long distance. To obtain the potential at all distances we follow the old

idea of interpolating the pQCD potential to the confining linear potential. There are many implementations of this idea, from the original Richardson potential to the recent one based on renormalon cancellation [16]. Our implementation is as follows. We combine the BR potential valid at short distances with the Cornell potential at long distance by matching them at an intermediate distance r_0 by

$$V_{\mathrm{BR}}^{(0)}(r_0) = V_{\mathrm{Cornell}}(r_0)\,, \quad \frac{d}{dr}V_{\mathrm{BR}}^{(0)}(r_0) = \frac{d}{dr}V_{\mathrm{Cornell}}(r_0)\,, \tag{10}$$

where

$$V_{\mathrm{Cornell}}(r) = a + \frac{b}{r} + cr\,. \tag{11}$$

Our choice of the Cornell potential as the long distance potential is based not only on its phenomenological success but its capability to fit the lattice data well [19]. We take the linear slope as $c = 0.18\ \mathrm{GeV}^2$, which is motivated by the lattice calculation [20]. Note that the matching equations completely determine the two free parameters in the Cornell potential. Thus our hybrid singlet potential reads

$$V^{(0)} = \begin{cases} V_{\mathrm{BR}}^{(0)}(r)\,, & r \leq r_0\,, \\ V_{\mathrm{Cornell}}(r)\,, & r \geq r_0\,. \end{cases} \tag{12}$$

The potential obtained this way is continuous and smooth (see, Fig. 1) at the matching point r_0.

The Borel resummed potential in pure QCD ($n_f = 0$) agrees well with lattice calculations to distances as large as $r \approx (700\mathrm{MeV})^{-1}$ [12]. Assuming that this behavior does not change drastically as the light quarks are introduced, we shall take the matching scale to be $r_0^{-1} = 1$ GeV. It will be shown that the dependence of the $\Upsilon(1S)$ binding energy on the matching scale is very small.

As we shall see in the next section the incorporation of the long distance confining potential is important even for such a spatially small system as $\Upsilon(1S)$, whose wave function turns out to have a sizable support where the difference between the confining potential and the perturbative potential is substantial. This implies that $\Upsilon(1S)$ is not completely perturbative even without the ultrasoft effects that will be discussed in Sec. IV.

III. SYSTEMATIC ERROR IN PURE PERTURBATIVE APPROACH

Recently there have been higher order calculations of the $\Upsilon(1S)$ spectrum within pure pQCD framework (see for example, [15, 21, 22, 23]). In this approach the leading order binding energy can be obtained by solving the Schroedinger equation of the Hamiltonian, obtained by the perturbative expansion of $2m_{pole} + V^{(0)}(r)$ in H_0 using the power expansions (6) at a fixed renormalization scale μ. Because of the renormalon cancellation between $2m_{pole}$ and $V^{(0)}$ the perturbative expansion of the spectrum is free from renormalon divergence (note that, throughout the paper, divergences due to the subleading renormalons are ignored). However, this approach can have a significant systematic error that cannot be seen from the convergence of the perturbative expansion or the renormalization scale dependence of the computed spectrum, since $2m_{pole} + V^{(0)}(r)$, although converges perturbatively, cannot produce the confining potential however high order it has been calculated.

To test whether the incorporation of the confining potential is important for $\Upsilon(1S)$ state, we calculate the ground state energy of H_0 in BR as well as the perturbative schemes
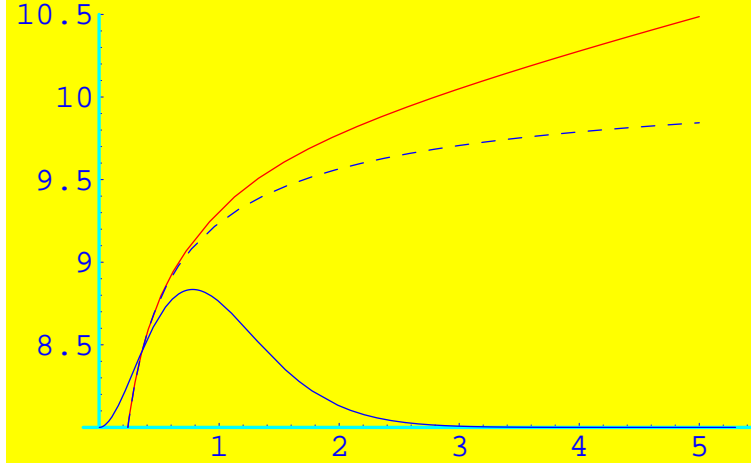
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FIG. 2: The total energy in BR method (solid line), pQCD method (dashed line) and zeroth order wave function $r^2 \Psi^{(0)}(r)^2$ shifted by 8 for displaying purpose.

TABLE I: The zeroth order total energy $E^{(0)}$ by pure perturbative expansion method with differing set of scale μ, the unit is in GeV.

μ	1	2	3	4	5
$E^{(0)}$	9.383	9.486	9.411	9.413	9.390

and compare them. We first plot $2m_{pole} + V^{(0)}(r)$ in Fig. 2. in both BR scheme and in pQCD using NNLO calculations of the expansions (6). We put $m_{\overline{\text{MS}}} = 4.2$ GeV and $\alpha_s(M_Z) = 0.118$ for both BR and pQCD schemes. In pQCD case the pole mass in the kinetic term were substituted by the NNLO pole mass in Eq. (6). The plot shows that the difference between these two schemes becomes substantial at $r \geq 1 \text{GeV}^{-1}$, where the wave function for $\Upsilon(1S)$ has significant support, and because of this the computed spectrum is sensitive on the employed method. We list the spectrum in Table I computed by the pQCD method. Since the BR result is $E^{(0)} = 9.574$ GeV, we find the energy difference, which depends on the renormalization scale μ, is at least 100 MeV. This implies that the achievable accuracy in an approach based on pure pQCD with a fixed renormalization scale is at best 100 MeV, and that an incorporation of the confining potential at long distance is essential for precision calculation of $\Upsilon(1S)$ spectrum.

IV. RECALCULATION OF THE LEUTWYLER-VOLOSHIN EFFECTS

The binding energy of the color singlet quarkonium is also affected by the emission and absorption of the ultrasoft gluons, which occur in pNRQCD through the singlet-octet-gluon vertex. This effect is nonperturbative and difficult to calculate. However, when the typical time scale of the quarkonium, $T \sim 1/mv^2$, is much smaller than the vacuum fluctuation scale, $1/\Lambda_{QCD}$, the quarkonium would 'see' the ultrasoft gluon as a constant random background field. The ultrasoft effect then becomes the Stark effect which in the case of quarkonium was first investigated by Voloshin [24] and Leutwyler [25].

Whether this Stark effect approximation is applicable to $\Upsilon(1S)$ is debatable, but nonethe-

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less we shall assume in the following it is the case. As argued in Ref. [26], even if this assumption fails, it is however likely that the Stark effect approximation will give an upper bound on the the ultrasoft effect, since the local condensate approximation gives an upper bound on the the two-point correlation function of the gluon field strength tensors.

According to Leutwyler and Voloshin the Stark effect can be large on $\Upsilon(1S)$ state and this is known to give a large uncertainty to the spectrum obtained in pQCD approach. The calculations by Leutwyler and Voloshin employed Coulombic potentials for the singlet as well as octet channels. Obviously, these potentials must not be good representations of the true potentials, and thus the validity of the calculations based on Coulombic potentials is questionable.

Our scheme of matching the Borel resummed potential at short distance with the confining potential at long distance is well-suited for the re-examination of this problem. In this section we recalculate the Stark effect using the hybrid singlet and octet potentials that were obtained by interpolations of the Borel resummed potential at short distance potentials and the confining potentials at long distance.

Surprisingly, our result, which will be given shortly, shows that the Stark effect in $\Upsilon(1S)$ state is very small. This is fortunate since it removes the large uncertainty coming from the ultrasoft nonperturbative effect.

The nonperturbative correction to the ground state energy is given by

$$\delta E_{NP} = \frac{\pi}{18} \langle \alpha_s G_{\mu\nu}^2 \rangle \sum_{i=1}^{3} \langle \Psi_0 | r^i \frac{1}{\hat{H}_0 - E_0} r^i | \Psi_0 \rangle \,, \tag{13}$$

where Ψ_0, E_0 are, respectively, the wave function and energy of the ground state of H_0, and $\hat{H}_0$ denotes the Hamiltonian for the octet channel:

$$\hat{H}_0 = 2m_{\mathrm{BR}} - \frac{\nabla^2}{m_{\mathrm{BR}}} + V_O(r) \,, \tag{14}$$

where $V_O(r)$ denotes the static octet potential.

With a recent preliminary calculation of the NNLO term [27] the octet potential is known perturbatively to NNLO. At short distance it is repulsive, but at long distance the octet potential is expected to have a confining linear potential. A hybrid octet potential that has good short distance behavior and a confining potential at long distance can be constructed in a similar fashion for the singlet potential. We first obtain the Borel resummed short distance potential employing the NNLO bilocal expansion, and then match it at the radius $r_0 = 1\,\mathrm{GeV}^{-1}$ to the fitted potential $V(r) = d + \sqrt{3.543 - 0.289\,r + 0.077\,r^2}$, which is obtained by fitting the lattice data for the octet potential [28]. The matching condition is that the potential be continuous at $r = r_0$, and this determines the constant d. Note that the asymptotic linear slope of the fitted potential $\sqrt{0.077} \approx 0.277\,\mathrm{GeV}^2$ is roughly consistent with the bag model calculation, according to which the slope of the octet potential is given by $\sqrt{7/4}$ times that of the singlet potential [29]. The octet potential obtained this way is plotted in Fig. 3. Now the nonperturbative correction Eq. (13) can be easily evaluated by numerically solving the equation

$$(\hat{H}_0 - E_0)|\chi_0^i> = r^i|\Psi_0> \,. \tag{15}$$

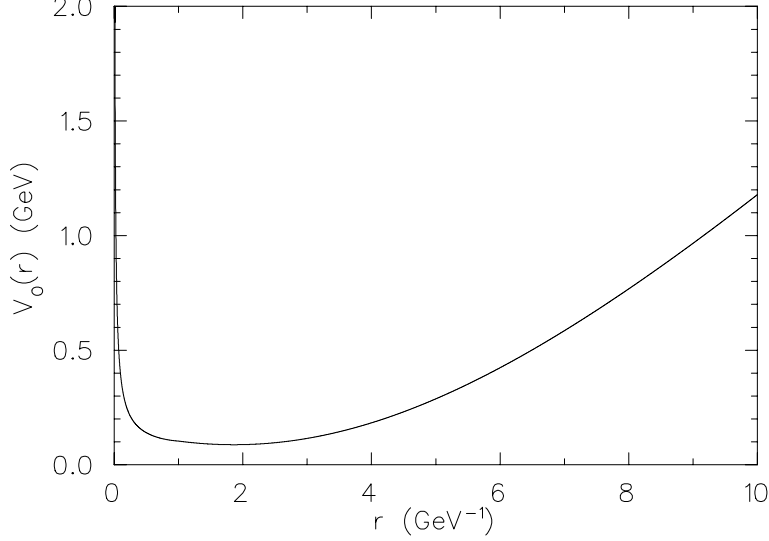FIG. 3: The octet potential, which is obtained by the Borel resummation at $r < 1$ GeV^{-1} and by fitting the lattice data at $r > 1$ GeV^{-1}.

V.   RESULTS

To obtain the binding energy we first solve the Schrödinger equation for H_0 numerically to obtain the zeroth order binding energy $E^{(0)}$ and the wave function $\Psi^{(0)}$, and treat the interactions V_{NP}, V_{hf} and V_{SI} as perturbations.

Before performing the calculation, we have to deal with two problems caused by our choice of the inverse of the interquark distance as the renormalization scale for the potential. The hyperfine and spin-independent operators in Eqs. (4, 5) contain the delta function $\delta^3(\vec{r})$, which will result in vanishing couplings $\alpha_s[1/(r \to 0)]$ in the numerator. In this case the inverse of the interquark distance as renormalization scale is clearly inappropriate. We solve this problem by noting that the renormalization scale for $V^{(0)}(r)$, $V^{(1)}(r)$ can be independently chosen from the rest terms in H' since these two terms should be RG invariant, independently from the rest terms of H'. We therefore choose the renormalization scale $\mu = 1/r$ for $V^{(0)}$ and V_{NA} and a fixed value of μ for the hyperfine and spin–independent terms. The second problem is caused by the value of $\alpha_s(1/r)$ at the opposite condition, when r is large. The coupling $\alpha_s(1/r)$ obtained by the perturbative β function is not reliable when r is large. To avoid this difficulty, one can use a simple regulation [30]

$$r\Lambda_{QCD} \to a \times \tanh\left(\frac{r\Lambda_{QCD}}{a}\right), \qquad (16)$$

in the running coupling $\alpha_s(1/r) \equiv f(r\Lambda_{QCD})$, which is obtained using the four-loop β function in $\overline{\text{MS}}$ scheme. The free parameter a should not be chosen too small a value so that the regularized coupling does not deviate too much from the perturbative coupling at short distances, nor too large a value to avoid an unphysical dip in the vicinity of the Landau pole. In our calculation, we set $a = 0.55$. We shall see that the computed $\Upsilon(1S)$ spectrum has only a mild dependence on this value. With this choice the regularized coupling changes little from the perturbative coupling when r is smaller than $r = 1$ GeV^{-1}, and at large r the regulation suppresses the growth of the perturbative coupling and approaches to an infrared
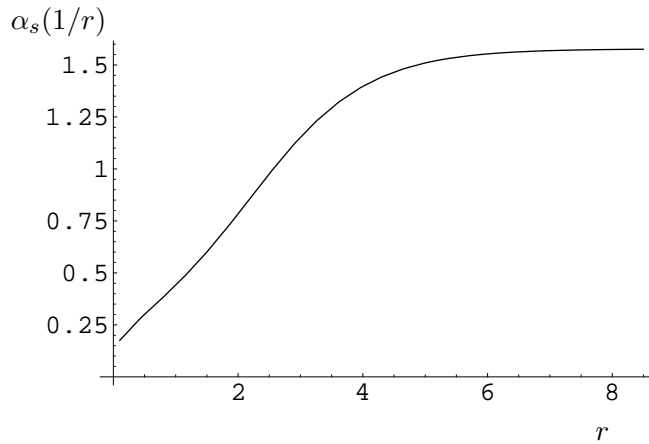
FIG. 4: Regularized $\alpha_s(1/r)$. In infrared region $\alpha_s(1/r)$ tends to a fixed value.

fixed value $\alpha_s(0) \approx 1.5$ (see Fig. 4). This infrared behavior of the regularized coupling is interesting, considering that the effective running couplings in various dispersive approaches display freezing behavior in the infrared limit [31]. Incidently, our $\alpha_s(0) \approx 1.5$ is very close to the predicted infrared fixed value $1/\beta_0 = 1.507$ (at $n_f = 4$) of the effective coupling in "analytic perturbation theory" [32].

Setting the input values of the strong coupling constant $\alpha_s(M_z) = 0.118$, we solve the non-relativistic Schrödinger equation numerically with the hybrid potential, obtain the zeroth order binding energy $E^{(0)}$ and wave function $\Psi^{(0)}$. We treat the interactions V_{NA}, V_{hf} and V_{SI} as perturbations and compute the perturbative corrections by calculating the expectation value

$$\delta E = \langle \Psi^{(0)} \mid (V_{NA} + V_{hf} + V_{SI}) \mid \Psi^{(0)} \rangle. \tag{17}$$

The contribution from each term is given in Table II. The values for the hyperfine and spin-independent contributions are with the renormalization scale $\mu = 3$ GeV.

The nonperturbative Leutwyler-Voloshin effect defined in Eq. (13) is computed by solving the equation (15). Taking the gluon condensate $\langle \alpha_s G^2_{\mu\nu} \rangle = 0.02 \pm 0.02$ we find $\delta E_{NP} = 5.8 \pm 5.8$ MeV, which is almost an order of magnitude smaller than estimates based on Coulombic potentials, which range $30 - -100$ MeV. We note that the smallness of this nonperturbative effect is very robust, depending little on the shape of the octet potential above the matching distance r_0. We also report, without details, that Leutwyler-Voloshin effect on the hyperfine splitting is very small, at most 1.6 MeV. This result indicates that the ultrasoft nonperturbative effect, which is regarded a major difficulty in precision computation of quarkonium spectrum, is small in $\Upsilon(1S)$ state.

Adding all these contributions as well as the charm mass effect, which shifts the binding energy about -20 ± 15 MeV [2], to fit the measured mass of $\Upsilon(1S)$, we obtain the bottom quark BR mass, $m_{\mathrm{BR}} = 4.901$ GeV, and the corresponding $\overline{\mathrm{MS}}$ mass $m_{\overline{\mathrm{MS}}} = 4.211$ GeV.

It is noteworthy that the hyperfine and spin–independent contributions cancel largely. The sum of these two contributions is $-20.5 + 30.4\alpha_s(\mu)$, in MeV, which shows that it has only a small renormalization scale dependence. Thus, our prediction of the b-quark mass depends little on the renormalization scale chosen for the hyperfine and spin–independent terms.

Our prediction of the hyperfine splitting can be found in Table III and Table IV. This result includes the leading as well as the subleading perturbative corrections for the hyperfine splitting [33]. At the leading order the hyperfine splitting is quite sensitive on the value of the renormalization scale, but when the subleading correction is included it has a very small renormalization scale dependence for $\mu \geq 2$ GeV. Taking $\mu = 3$ GeV and $\alpha_s(M_z) = 0.118$, and the subleading correction as the theoretical uncertainty, we estimate the hyperfine
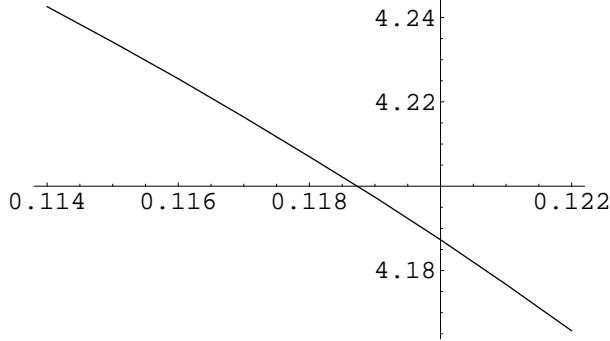
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FIG. 5: The extracted b-quark $\overline{\text{MS}}$ mass vs $\alpha_s(M_z)$.

splitting to be 50 ± 8 MeV.

In the following we discuss on the uncertainties on the extracted b-quark mass in our scheme. (i) Uncertainty of $\alpha_s(M_z)$: With different input values of $\alpha_s(M_z)$, the extracted BR mass as well as $\overline{\text{MS}}$ mass will change. Fig. 5 shows the estimates of b-quark $\overline{\text{MS}}$ mass against $\alpha_s(M_z)$. We can see from the figure, if the value of $\alpha_s(M_z)$ vary from 0.117 to 0.120, the $\overline{\text{MS}}$ mass will vary from 4.220 GeV to 4.192 GeV. (ii) Uncertainty of r_0: Varying the matching point r_0, at which the phenomenological potential is stitched to the perturbative BR potential, will cause some uncertainty. With about 10% variation of r_0, the extracted b quark $\overline{\text{MS}}$ mass varies only within 3 MeV. This uncertainty can be completely ignored. (iii) Uncertainty of λ: In our calculation, we chose the slope of the linear potential $\lambda = 0.18$ GeV^2. We shall assign 10% variation on λ, which causes the $\overline{\text{MS}}$ b-quark mass vary 4.211 ± 0.001. Again the error is small enough to be ignored. The $\Upsilon(1S)$ state is spatially small enough that its spectrum is not so sensitive on the shape of the confining potential. (iv) Uncertainty of the α_s regularization constant a: The regularization equation (16) of α_s involves the constant a, on which the infrared fixed value of $\alpha_s(1/r)$ depends strongly. For instance when we vary the value of a from 0.35 to 0.75, about 36% variation from the central value $a = 0.55$, the infrared fixed value of $\alpha_s(1/r)$ vary from 0.58 to 28.6, respectively. However, the variation of the extracted $\overline{\text{MS}}$ b-quark mass is negligibly small, only from 4.205 to 4.217. This is similar to the small dependence on the slope λ. Since only the long distance behavior of the regularized coupling is sensitive on a the spectrum of the $\Upsilon(1S)$ has only a small dependence on the value of a.

TABLE II: Contributions to $1S$ state in unit of MeV.

$E^{(0)}$	V_{SI}	V_{NA}	V_{hf}	$\delta E_{NP}^{(0)}$
9575	-66	-113	58	6

VI. SUMMARY

We investigated the $\Upsilon(1S)$ system using a hybrid Hamiltonian for heavy quarkonium, which was built by matching the Cornell potential at long distances to the Borel resummed

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pQCD potential at short distances. This matching fixes completely the free parameters of the Cornell potential except for the slope of the linear term. The hybrid Hamiltonian employs an infrared sensitive, all-order resummed pole mass and static potential, and is free from the inevitable scale mixing problem present in approaches based on short-distance quantities.

The incorporation of the long distance confining potential was shown to be essential for precision calculation of $\Upsilon(1S)$ system. The $\Upsilon(1S)$ is not a completely perturbative system, even without the ultrasoft contributions. We note that pure pQCD calculations of $\Upsilon(1S)$ spectrum at a fixed renormalization scale should contain a sizable systematic error of about 100 MeV.

The nonperturbative ultrasoft effects were reconsidered in the modeling of Leutwyler and Voloshin, employing the hybrid singlet and octet potentials. We find these effects are very small, contributing at most 6 MeV to the $\Upsilon(1S)$ spectrum, and negligible contribution to the hyperfine splitting which was found to be 50 ± 8 MeV at $\alpha_s(Mz) = 0.118$.

TABLE III: The hyperfine splitting (leading order and next to leading order) vs $\alpha_s(M_z)$ at $\mu = 3$ GeV, the unit is in MeV.

$\alpha_s(M_z)$	0.116	0.117	0.118	0.119	0.120
V_{hf}	49	54	58	63	69
δV_{hf}	-7	-7	-8	-9	-10

TABLE IV: The numerical results with different set of μ at $\alpha_s(M_z) = 0.118$, the unit is in MeV.

μ	1	2	3	4	5
V_{hf}	101	69	58	52	49
δV_{hf}	-81	-21	-8	-3	0.4
$V_{hf} + \delta V_{hf}$	20	48	50	50	49
V_{SI}	-101	-75	-66	-61	-58
$m_{\overline{\text{MS}}}$	4.207	4.210	4.211	4.212	4.212

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